Mission-Function Control for Slew Maneuver of a Flexible Space Structure

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A new control algorithm, named the mission-function control method, is introduced and applied to slew maneuver of a spacecraft with a flexible appendage. The control algorithm is based upon the Lyapunov method applied to a mechanical system combined with a control system and employs the concept of generalized energy functions. The flexible appendage is modeled in terms of the partial differential equations that are believed to describe the distributed systems most precisely. Implementation of the control algorithm naturally results with physical meaning, and it is shown that sensing of the bending moment and shearing force at the root of the flexible appendage is essential. Results of numerical simulation show an excellent controlled behavior for the slew maneuver.

I. Introduction

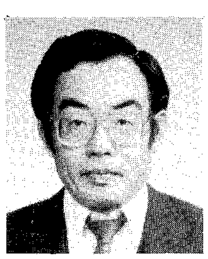
MANY challenging aspects must be clarified to control motion of the large space structures (LSS) that are growing in size with increasing demands on their precise pointing. Included among these are accurate modeling techniques for LSS, which are distributed systems with uncertain parameters. A coordinated approach is needed from both of the twin disciplines of dynamics and control to manage a large number of degrees of freedom that is theoretically infinite for LSS, and appropriate numbers and locations of sensors/actuators that result in a hybrid system with distributed mass and discretized sensors/actuators.

With these aspects in mind, this paper presents a new control algorithm for the slew maneuver in planar motion of a spacecraft consisting of a rigid central body and a flexible appendage attached to it. The slew maneuver is one of the necessary functions for some advanced spacecraft changing their attitude angle from one position to a desired attitude. In this maneuver, vibration of the flexible appendage is naturally excited and excessive excitation of the vibration is undesirable. Thus, it becomes necessary to control the slew maneuvers generating a large-angle attitude reorientation while minimally suppressing the excited vibration of the flexible appendage and eliminating it at the end of the maneuver.

The slew maneuver generally is defined to be the solution of an optimal two-point boundary-value problem, given the initial and final states prescribed in a finite time.[1] This leads to the application of standard optimal control theories, including such approaches as the linear quadratic Gaussian or the calculus of variation, combined with a modal decomposition of flexible bodies. Many excellent papers seek to solve the problem using the Pontryagin's principle,[2,3] the calculus of variation,[4] the linear-quadratic Gaussian regurator,[5–7] and the shaped-torque technique.[8] The modal decomposition involves inherent difficulty, since the control algorithm must manage a large number of degrees of freedom and this results in difficulties in solving the two-point boundary-value problem. This fact suggests one seek to manage the dual (rigid and elastic) modes independently[9] or to describe the distributed-parameter system in terms of partial differential equations,[10] although one usually encounters difficulty through application of standard control algorithms.

Several papers are devoted to examine the laboratory implementations and the practical validity of the control algorithm for the slew maneuver by hardware experiments.[9,11–13]

This paper considers the control problem of the slew maneuver as generating a large-angle attitude reorientation while



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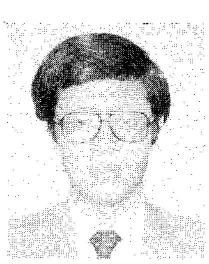

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minimally suppressing the excited vibration of the flexible appendage and driving it and the reorientation deviation into an allowable small region at the end of the maneuver, or in other words, the finite time constraint is removed as in Ref. 5. This is because the preceding approach with the finite terminal time constraint usually gives the open-loop optimal control and any disturbance or uncertainty of the system model is likely to cause unsatisfactory system response.

Frequently noted are too sophisticated applications of the control theories to the control design of LSS in which it is difficult to retain the physical meaning of the parameters.[14] A new control algorithm presented here is named the "mission-function control" and is one type of the Liapunov method.[15] The algorithm is introduced through inspection on the mechanical features of LSS and uses concepts of generalized energy functions including the mechanical and potential energies of the flexible structure. It is an approach involving the coordination of both the mechanics and control professional fields. The algorithm is therefore naturally able to treat the partial differential equations as a most precise description of distributed systems such as LSS and thus suffers no spillover effect. Furthermore, the values to be sensed through the control are naturally derived to be the bending moment and shearing force at the root of the flexible appendage added with attitude angle and its velocity while an actuator is supposed a priori to be a torque applied to the rigid central body of the spacecraft.

Results of numerical simulation show the usefulness of the present algorithm applied to the slew maneuver of a spacecraft with a flexible appendage.

II. System Model

Illustrated in Fig. 1 is a model of a spacecraft consisting of a rigid central body and a flexible appendage attached to it, where the appendage is assumed to be a beam with one end fixed to the rigid central body and the other end free. Rotational and vibrational motions are confined to the plane of the figure. The only actuator input available to implement control is a torque T_r applied to the rigid central body through any torquer placed in it. The center of mass of the spacecraft is assumed to always remain at C for simplicity of the analysis. The flexible appendage is modeled as the Bernoulli-Euler beam.

The structural damping is a very important factor in the design of the controller for the LSS, and it may seem necessary to include for consideration the dissipation model into the equations of motion of the flexible structure. This paper employs two types of the dissipation models to treat with the Bernoulli-Euler beam model; one represents the dissipation force proportional to the velocity of the beam deflection, $\partial v/\partial t$; and the other represents the dissipation force proportional to the strain rate, $(\partial/\partial t)(\partial^4 v/\partial u^4)$. However, an accurate assessment of energy dissipation is beyond the scope of this paper.[16]

The partial differential equations describe the motion for the present system if higher-order terms are neglected as follows:

$$I_r(\partial^2\theta/\partial t^2) + \int_{l_0}^{l} \rho u[\partial^2 v/\partial t^2 + u(\partial^2\theta/\partial t^2)]\,\mathrm{d}u = T_r \quad (1)$$

$$\rho\left(\frac{\partial^2 v}{\partial t^2} + u\frac{\partial^2\theta}{\partial t^2}\right) + c_1\frac{\partial v}{\partial t} + c_4\frac{\partial^5 v}{\partial t\partial u^4} + \frac{\partial^2}{\partial u^2}\left(EI\frac{\partial^2 v}{\partial u^2}\right) = 0 \quad (2)$$

with the boundary conditions

$$v = \partial v/\partial u = 0 \quad \text{at} \quad u = l_0 \quad (3a)$$

$$\partial^2 v/\partial u^2 = \partial v^3/\partial u^3 = 0 \quad \text{at} \quad u = l \quad (3b)$$

where I_r and T_r denote the moment of inertia and a control torque, respectively, of the central body, and ρ, EI, c_1, and c_4 denote the line density, the bending rigidity, the structural damping coefficients for the viscous damping and for the Kelvin-Voight damping, respectively, of the flexible appendage assumed to be a Bernoulli-Euler beam model cramped at the root ($u = l_0$) and free at the tip ($u = l$), and other notations are shown in Fig. 1.

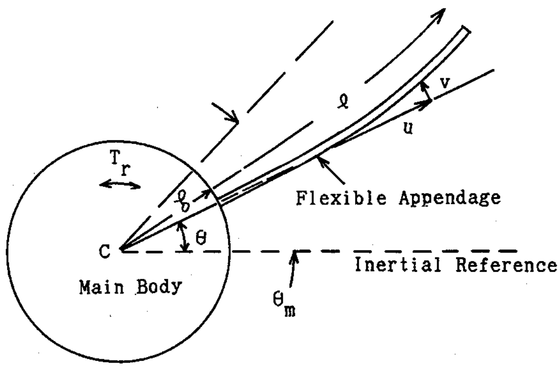

Fig. 1 A spacecraft model for the slew maneuver.

It may be noted that pertinent higher-order terms have been neglected in the equations of motion given by Eqs. (1) and (2). Analysis for the present control algorithm is given in the Appendix where certain higher-order terms are included in the equations of motion.

The Hamiltonian H of the system is described as

$$H = \frac{1}{2}I_r\left(\frac{\partial\theta}{\partial t}\right)^2 + \frac{1}{2}\int_{l_0}^{l}\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial\theta}{\partial t}\right)^2 \mathrm{d}u + \frac{1}{2}\int_{l_0}^{l} EI\left(\frac{\partial^2 v}{\partial u^2}\right)^2 \mathrm{d}u \quad (4)$$

and its derivative with respect to time is easily shown to be

$$\frac{\mathrm{d}H}{\mathrm{d}t} = T_r\frac{\partial\theta}{\partial t} - c_1\int_{l_0}^{l}\left(\frac{\partial v}{\partial t}\right)^2 \mathrm{d}u - c_4\int_{l_0}^{l}\left(\frac{\partial^3 v}{\partial t\partial u^2}\right)^2 \mathrm{d}u \quad (5)$$

III. Mission-Function Control Applied to Slew Maneuver

The slew maneuver seeks to change the attitude angle θ from initial state θ_0 to any desired angle θ_m leaving the vibration of the flexible appendage inactive at the end of the maneuver. This maneuver can be defined by the following mission: The mission is to change the dynamical state of the system from the initial state with $\theta = \theta_0$ into a desired state (call it the mission state) with

$$\partial\theta/\partial t = 0 \quad (6a)$$

$$\theta = \theta_m \quad (6b)$$

$$\frac{1}{2}\int_{l_0}^{l}\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial\theta}{\partial t}\right)^2 \mathrm{d}u + \frac{1}{2}\int_{l_0}^{l} EI\left(\frac{\partial^2 v}{\partial u^2}\right)^2 \mathrm{d}u = 0 \quad (6c)$$

where Eq. (6c) denotes the sum of the kinematic energy and the elastic strain energy, respectively, of the flexible appendage.

A control algorithm is sought through selection of a mission function M that is positive definite and is zero only at the mission state, Eq. (6). The mission function M is specified in this case as

$$2M = a_1 I_r\left(\frac{\partial\theta}{\partial t}\right)^2 + a_2(\theta - \theta_m)^2 + (a_1 + b) \times\left[\int_{l_0}^{l}\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial\theta}{\partial t}\right)^2 \mathrm{d}u + \int_{l_0}^{l} EI\left(\frac{\partial^2 v}{\partial u^2}\right)^2 \mathrm{d}u\right] \quad (7a)$$

or

$$M = a_1 H + \frac{1}{2} a_2(\theta - \theta_m)^2 + \frac{1}{2} b\left[\int_{l_0}^{l} \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial \theta}{\partial t}\right)^2 du + \int_{l_0}^{l} EI\left(\frac{\partial^2 v}{\partial u^2}\right)^2 du\right] \tag{7b}$$

where $a_i > 0$ $(i = 1,2)$ and b $(b + a_1 > 0)$ are weighting coefficients. It may be noted that the mission function contains the generalized energy functions in terms of $\partial\theta/\partial t$, θ, and the energy functions of the flexural motion of the flexible appendage.

Using Eqs. (1–3) and assuming that the bending rigidity EI and the damping coefficients c_1 and c_4 are constant through the beam, one obtains the derivative of the mission function with respect to time as

$$\frac{dM}{dt} = \tilde{T}\frac{\partial \theta}{\partial t} - c_1(a_1 + b)\int_{l_0}^{l}\left[\frac{\partial v}{\partial t} + \frac{b}{2(a_1+b)}u\frac{\partial\theta}{\partial t}\right]^2 du - c_4(a_1+b)\int_{l_0}^{l}\left(\frac{\partial^3 v}{\partial t \partial u^2}\right)^2 du \tag{8}$$

where

$$T_r = (1/a_1)\{\tilde{T} - a_2(\theta - \theta_m) + b(M_0 - l_0 S_0) - [c_1 b^2/4(a_1+b)][(l^3 - l_0^3)/3](\partial\theta/\partial t) + (c_4 b/EI)(\partial M_0/\partial t - l_0 \partial S_0/\partial t)\} \tag{9}$$

$\tilde{T}$ is the freely assignable part of the control force T_r and M_0 and S_0 denote the bending moment and the shearing force, respectively, at the root of the flexible appendage:

$$M_0 = EI(\partial^2 v/\partial u^2)\,|\,l_0, \quad S_0 = EI(\partial^3 v/\partial u^3)\,|\,l_0 \tag{10}$$

The mission function is positive definite and is zero when the mission state is obtained. The mission-function control is to reduce the value of the mission function, i.e., the time derivative of the mission function, Eq. (8), is forced to be negative definite through the control process. This can be obtained by choosing $\tilde{T}$, for example, as

$$\tilde{T} = -k(\partial\theta/\partial t) \qquad [k > 0] \tag{11}$$

and the derivative of the mission function with respect to time is obtained as follows:

$$\frac{\partial M}{\partial t} = -k\left(\frac{\partial\theta}{\partial t}\right)^2 - c_1(a_1+b)\int_{l_0}^{l}\left[\frac{\partial v}{\partial t} + \frac{b}{2(a_1+b)}u\frac{\partial\theta}{\partial t}\right]^2 du - c_4(a_1+b)\int_{l_0}^{l}\left(\frac{\partial^3 v}{\partial t\partial u^2}\right)^2 du \tag{12}$$

which is negative definite.

Thus, it is evident that the mission state is asymptotically stable and the control torque T_r will accomplish the mission if implemented through Eqs. (9) and (11) as follows:

$$T_r = (1/a_1)\{a_2(\theta_m - \theta) - k(\partial\theta/\partial t) + b(M_0 - l_0 S_0) - c_1[b^2/4(a_1+b)][(l^3 - l_0^3)/3](\partial\theta/\partial t) + c_4(b/EI)(\partial M_0/\partial t - l_0\partial S_0/\partial t)\} \tag{13}$$

Note that simple reduction of T_r is given as

$$T_r = (1/a_1)\{a_2(\theta_m - \theta) - k(\partial\theta/\partial t) + b(M_0 - l_0 S_0) - c_1 b\int_{l_0}^{l}(\partial v/\partial t)u\, du + c_4(b/EI)(\partial M_0/\partial t - l_0 \partial S_0/\partial t)\} \tag{14}$$

The expression of T_r in Eq. (13) is derived without including such information of a distributed nature as $\int_{l_0}^{l}(\partial v/\partial t)u\, du$ which is hard to implement.

It should be noted that Eq. (13) contains values to be sensed as θ, $\partial\theta/\partial t$, M_0, and S_0 (and $\partial M_0/\partial t$ and $\partial S_0/\partial t$ when $c_4 \neq 0$), and it is the naturally derived result that motion of a flexible appendage is essentially sensed by the bending moment and the shearing force at the root of the beam. It also may be noted that the sensing is not done as usual through the acceleration but the force, since the control is delivered through the force and the relation that force = mass × acceleration is embedded not within the sensor/actuator implementation but within the analytical procedure.

IV. Theoretical Examination

The theoretical aspect of the mission-function control is treated in Ref. 15, and its special feature is examined for the current slew maneuver.

Consider a system S that consists of two subsystems S_1 with control force and S_2 without control force and is described by the following equations:

$$S = \begin{cases} S_1\colon\ \ddot{x} + f(x,\dot{x},y,\dot{y}) = u & (15) \\ S_2\colon\ \ddot{y} + g(x,\dot{x},y,\dot{y})\dot{x} + h(x,y) + l(\dot{y}) = 0 & (16) \end{cases}$$

where x and y are the state vectors, u denotes the control force, and $(\dot{\ })$ denotes derivative with respect to time.

If for the system $\dim y \leqslant \dim x$, $\dot{y}^T l(\dot{y}) \geqslant 0$, and there exists a function $V(x,y) \geqslant 0$ satisfying $h(x,y) = \partial V(x,y)/\partial y$, then a mission function M can be defined as follows:

$$M = (1/2)a_1\dot{x}^T\dot{x} + (1/2)a_2 x^T x + (1/2)\dot{y}^T\dot{y} + V(x,y) \tag{17}$$

where $(\ )^T$ denotes the transposition.

The function h can be regarded as a differential operator for the present slew problem and Eqs. (15) and (16) can be reduced, respectively, to the following expressions:

$$S_2\colon\ \ddot{y} + g(x,\dot{x},y,\dot{y})\dot{x} + h(y) + l(\dot{y}) + m\ddot{x} = 0 \tag{18}$$

$$M = (1/2)a_1\dot{x}^T\dot{x} + (1/2)a_2 x^T x + (1/2) \times (\dot{y} + m\dot{x})^T(\dot{y} + m\dot{x}) + V(y) \tag{19}$$

Lemma: For the system S, there exists a feedback control algorithm $u = u(x,\dot{x},y,\dot{y})$ that satisfies the condition $\partial M/\partial t \leqslant 0$. (The proof is evident.)

V. Numerical Simulation

The numerical simulation employs data from the LSS laboratory model at the National Aerospace Laboratory.[9] The parameters of the model are $I_r = 8.67$ Nms^2, $l_0 = 0.08$ m, the flexible appendage is a simple aluminum beam of the size $1.5 \times 0.1 \times 0.003$ m, $l = 1.585$ m, the line density per meter $\rho = 8.073$ Nm^{-2}s^2, and the bending rigidity $EI = 15.45$ Nms^2. The mission angle is set to be $\theta_m = 36$ deg for the present simulation.

A. Simulation Using Difference Equations

The equations of motion, Eqs. (1) and (2), are modified into finite-difference equations for solution by the numerical method.[17]

Figures 2–4 show representative results of the numerical simulation. Each figure contains four figures from the top to the bottom denoting the attitude angle θ, the tip deflection Y_{tip} of the beam, the shearing force S_0, and the bending moment M_0 at the root of the beam, respectively, against time.

A case is shown in Fig. 2 when $b = k = 0$, and it is found that no mission-function control is applied except for the control of $\partial\theta/\partial t$ and $\theta - \theta_m$ ($a_1 = 1.0$, $a_2 = 0.5$). On the other

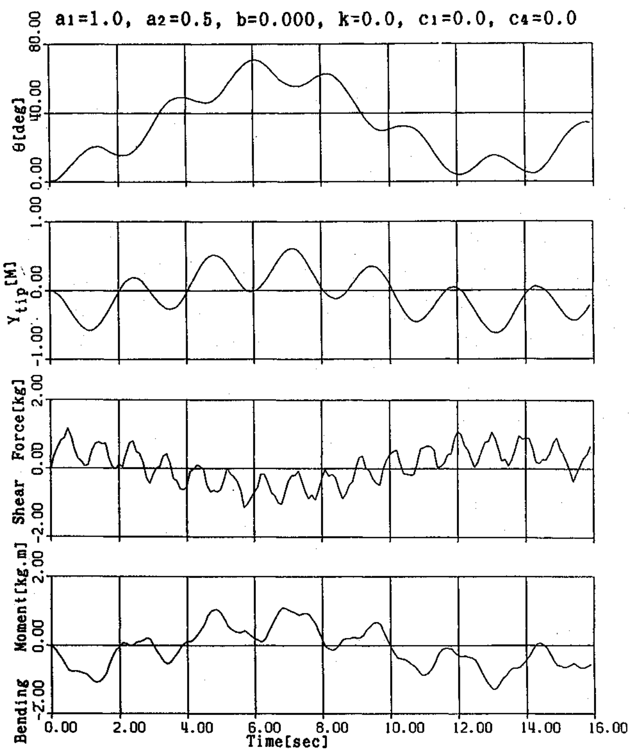

Fig. 2 System response ($a_1 = 1.0$, $a_2 = 0.5$, $b = k = c_1 = c_2 = 0.0$).

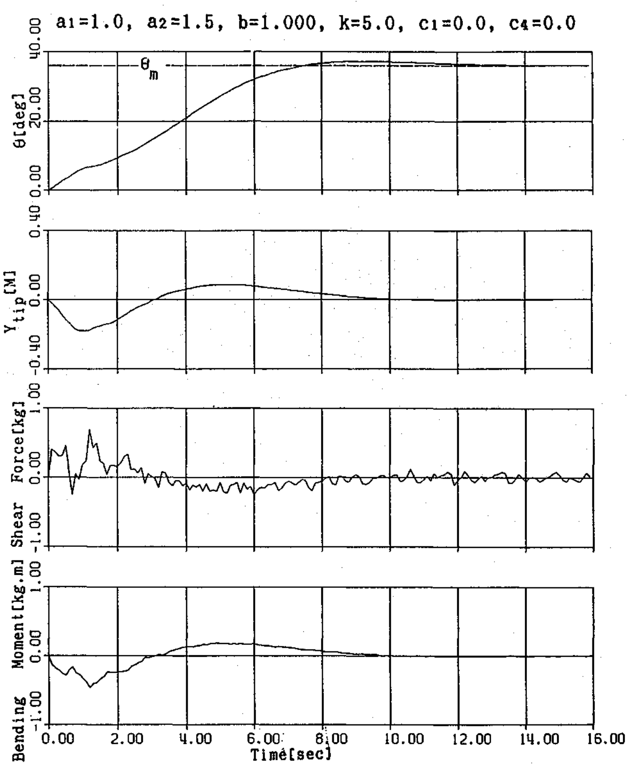

Fig. 3 System response ($a_1 = 1.0$, $a_2 = 1.5$, $b = 1.0$, $k = 5.0$, $c_1 = c_4 = 0.0$).

hand, Fig. 3 shows the results when $a_1 = 1.0$, $a_2 = 1.5$, $b = 1.0$, and $k = 5.0$, and this case shows that the present control algorithm works very well. A case when the value of b is increased to 2.0 is shown in Fig. 4. From the comparison of Fig. 4 with Fig. 3, it may be noted that weighting on the vibrational suppression of the flexible appendage is increased and suppression of vibration of the beam is better, but the value of θ approaches slowly to θ_m, respectively, than those in Fig. 3.

The effect of external and internal damping is shown in Fig. 5 with $c_1 = 0.01$ and in Fig. 6 with $c_4 = 0.01$, respectively. These figures do not show particular improvement of the slew performance with respect to that shown in Fig. 3 with $c_1 = c_2 = 0$, since the value of the damping coefficients are small as usual and only rather smooth variation of M_0 and S_0 may be noted.

B. Modal Decomposition

It is interesting to examine the behavior of the controlled system using the modal decomposition employed most generally in the analysis of the LSS motion. Figures 7–10 present root-locus plots for the present system where the imaginary axis is stretched for only the rigid mode.

The effect of the value of k increasing from $k = 0.2$–6.0 is shown in Fig. 7, where the small white circles denote the characteristic roots of the rigid mode of the spacecraft and the first to the fourth modes of the flexible appendage, respectively, when the present mission-function control is not applied. It is shown in Fig. 7 that increase in the value of k causes improved performance of the controlled system, and this is dominant for the rigid mode and the lower modes. The effect of increasing the value of b is shown in Fig. 8, where the symbol × denotes the characteristic roots for values of the weighting coefficients $a_1 = 1.0$, $a_2 = 1.5$, $b = 0.2$, and $k = 5.0$. The parameter that is varied is the weighting coefficient b for the vibrational motion of the flexural appendage, and it is observed that the performance for the vibrational suppression of the flexible modes improved but that of the rigid modes deteriorate as the value of b increases, since the weighting for

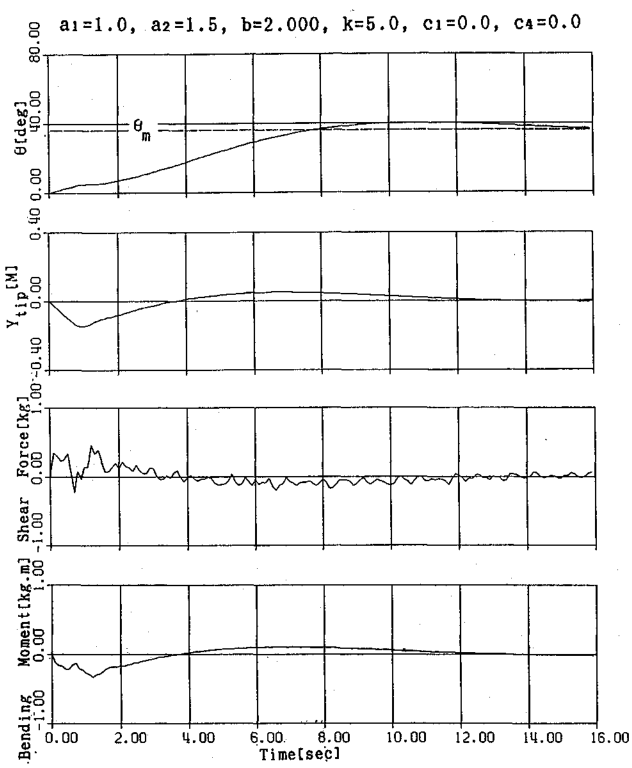

Fig. 4 System response ($a_1 = 1.0$, $a_2 = 1.5$, $b = 2.0$, $k = 5.0$, $c_1 = c_4 = 0.0$).

the flexible modes relatively increases the total energy of the flexible appendage. This fact confirms the results shown in comparison between Figs. 3 and 4. It also may be noted that no spillover effect is observed although it is clear since the present algorithm employs the partial differential equations as the model description.

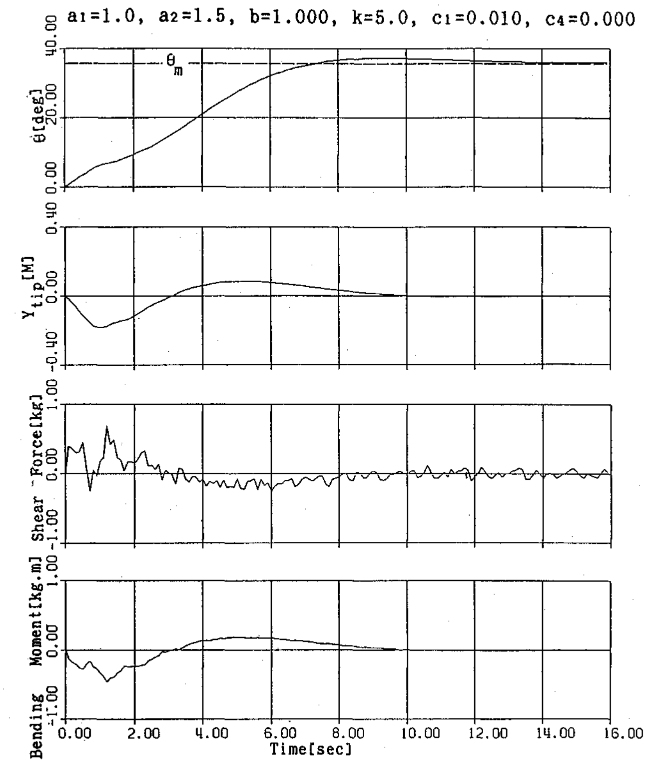

Fig. 5 System response ($a_1 = 1.0$, $a_2 = 1.5$, $b = 1.0$, $k = 5.0$, $c_1 = 0.01$, $c_4 = 0.0$).

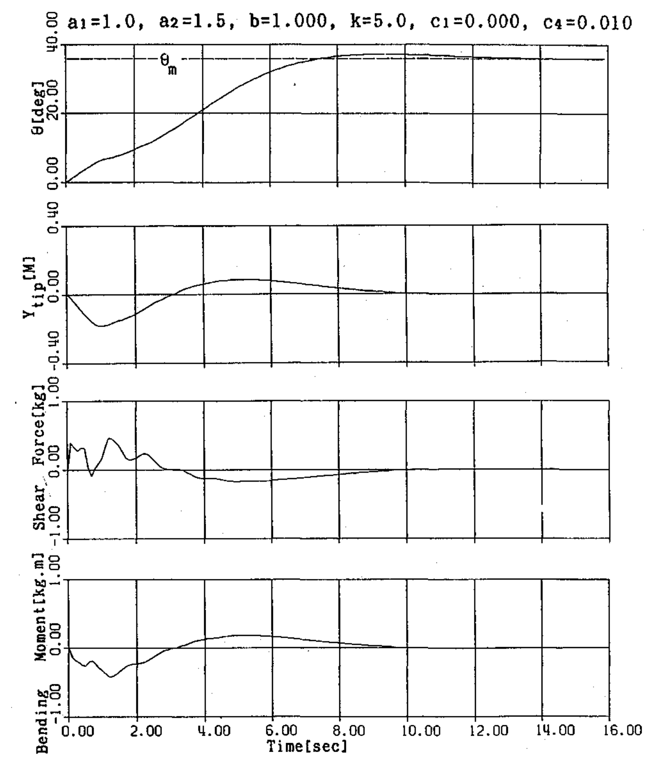

Fig. 6 System response ($a_1 = 1.0$, $a_2 = 1.5$, $b = 1.0$, $k = 5.0$, $c_1 = 0.0$, $c_4 = 0.01$).

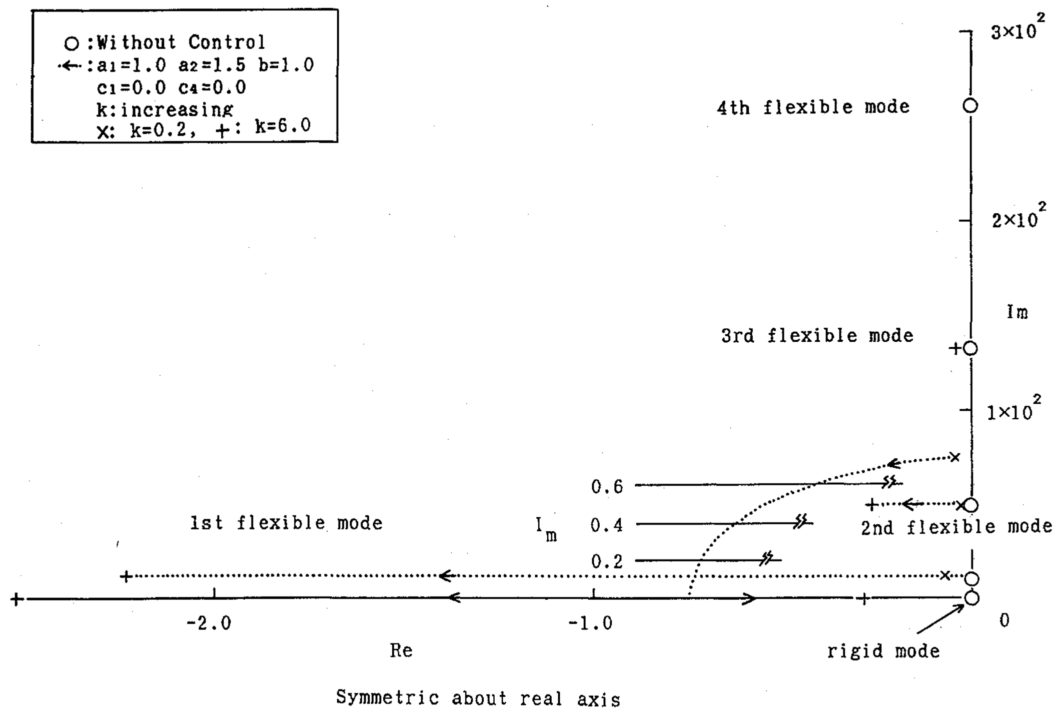

Fig. 7 Root loci for the slew maneuver. (The parameter varied is k and other weighting coefficients are constant.)

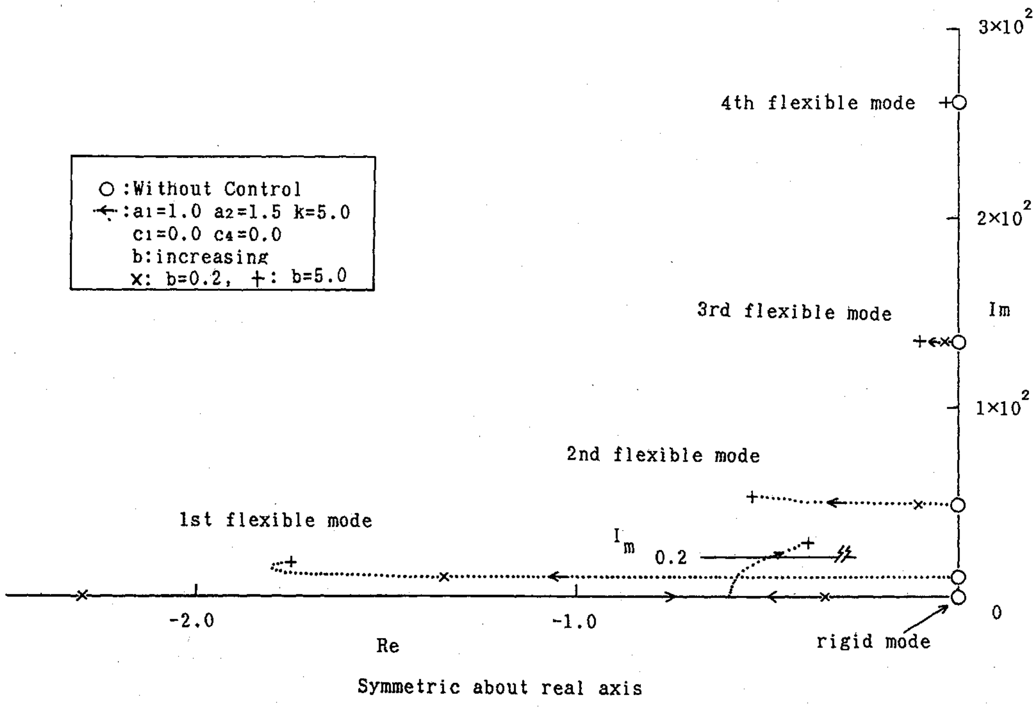

Fig. 8 Root loci for the slew maneuver. (The parameter varied is *b* and other weighting coefficients are constant.)

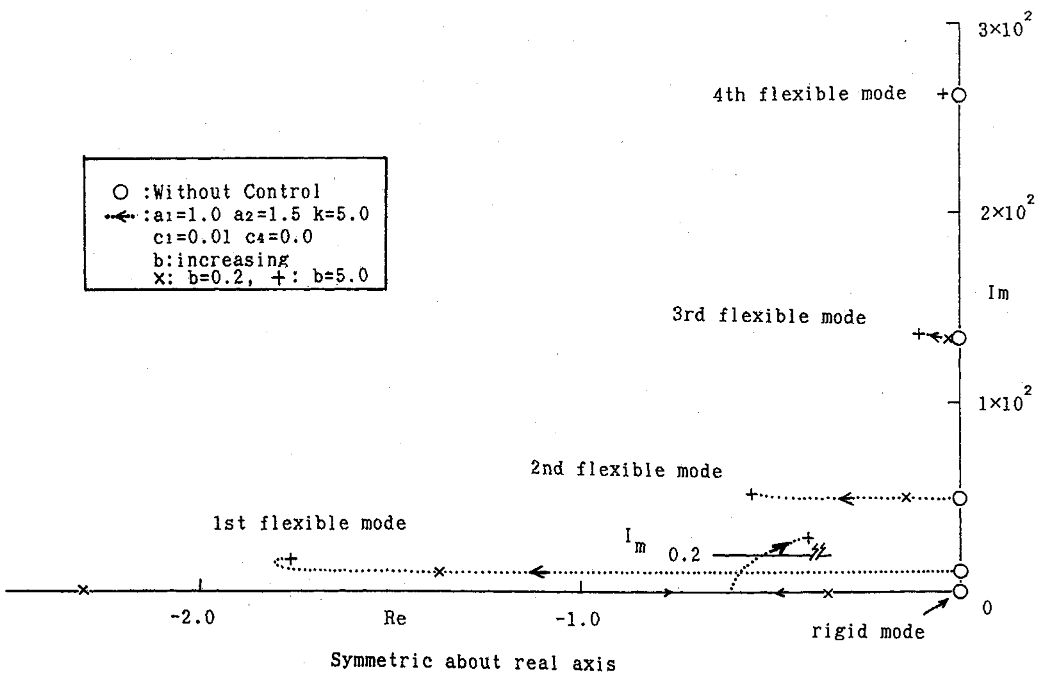

Fig. 9 Root loci for the slew maneuver. (The parameter varied is *b* and other weighting coefficients are constant and $c_1 = 0.01$.)

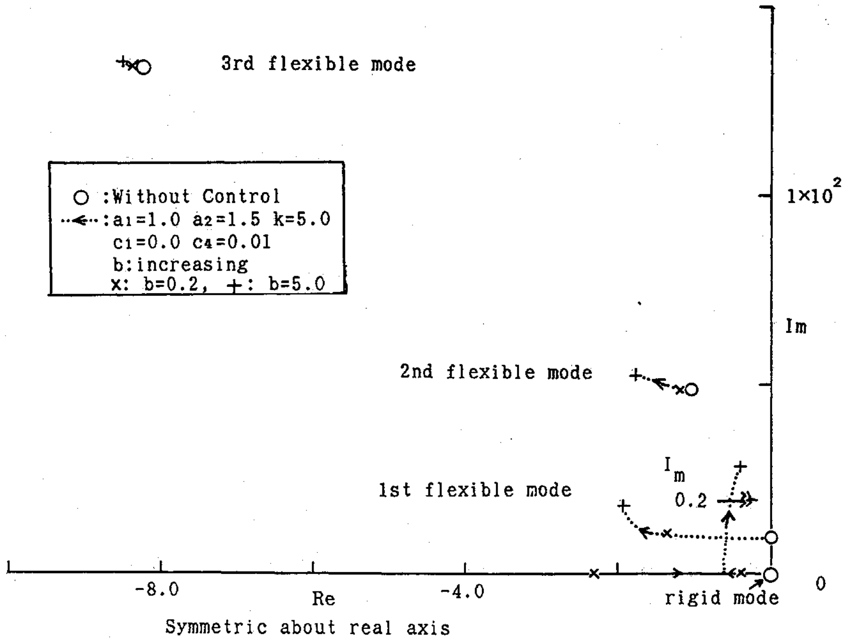

Fig. 10 Root loci for the slew maneuver. (The parameter varied is b and other weighting coefficients are constant and $c_4 = 0.01$.)

Figures 9 and 10 show the case when the damping coefficients $c_1 = 0.01$ and $c_4 = 0.01$, respectively. The effect of this dissipation is manifested as an improvement in the performance of the modes or as a shift of the root loci to the left as shown in these figures.

VI. Conclusion

A new control algorithm, the mission-function control method, is presented with application to the slew maneuver of a spacecraft with a flexible appendage. The following conclusions are obtained:

1) The system being treated is described mathematically by the partial differential equations that are believed to model most precisely the distributed systems such as the LSS.

2) Attention is focused on the dynamical features of the flexible structure such as its energy functions through procedure of reduction of the algorithm.

3) The control algorithm is regarded as a type of Liapunov method for a mechanical system combined with the control system, and its preferable features are dominant such as the robustness of the optimal regulator since no spillover effect is observed.

4) The algorithm naturally manifests the attractive features that the values to be sensed are essentially the attitude angle θ, its rate $\partial\theta/\partial t$, the bending moment M_0, and the shear force S_0 (and $\partial M_0/\partial t$ and $\partial S_0/\partial t$ when $c_4 \neq 0$, respectively) at the root of the flexible appendage. This sensor location and number means easy and practical implementation of the present control.

Results from the numerical simulation proved excellent controlled slew maneuver employed with the mission-function control algorithm.

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Appendix: Analysis of the Algorithm When the Higher-Order Terms of Dynamics are Included

It may be noted that higher-order terms of dynamics have been neglected in the equations of motion given by Eqs. (1) and (2). Analysis for the present control algorithm is given here when the higher-order terms are included in the equations of motion.

The equations of motion given by Eqs. (1) and (2) are described as follows when the higher-order terms arise from the centripetal and tangential accelerations due to rotation of the central body:

$$I_r\frac{\partial^2\theta}{\partial t^2} + \int_{l_0}^{l}\rho u\left(\frac{\partial^2 v}{\partial t^2} + u\frac{\partial^2\theta}{\partial t^2}\right)\mathrm{d}u + \frac{\partial}{\partial t}\times\left\{\frac{\partial\theta}{\partial t}\int_{l_0}^{l}\rho\left[v^2 - \frac{l^2-u^2}{2}\left(\frac{\partial v}{\partial u}\right)^2\right]\right\}\mathrm{d}u = T_r \tag{A1}$$

$$\rho\left(\frac{\partial^2 v}{\partial t^2} + u\frac{\partial^2\theta}{\partial t^2}\right) + \rho\left(\frac{\partial\theta}{\partial t}\right)^2\left[\frac{\partial}{\partial u}\left(\frac{l^2-u^2}{2}\frac{\partial v}{\partial u}\right) - v\right] + c_1\frac{\partial v}{\partial t} + c_4\frac{\partial^5 v}{\partial t\partial u^4} + \frac{\partial^2}{\partial u^2}\left(EI\frac{\partial^2 v}{\partial u^2}\right) = 0 \tag{A2}$$

where the boundary conditions are the same as Eq. (3).

The Hamiltonian H of this case is described as

$$H = \frac{1}{2}\left\{I_r + \int_{l_0}^{l}\rho\left[v^2 - \frac{l^2-u^2}{2}\left(\frac{\partial v}{\partial u}\right)^2\right]\mathrm{d}u\right\}\left(\frac{\partial\theta}{\partial t}\right)^2 + \frac{1}{2}\int_{l_0}^{l}\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial\theta}{\partial t}\right)^2\mathrm{d}u + \frac{1}{2}\int_{l_0}^{l}EI\left(\frac{\partial^2 v}{\partial u^2}\right)^2\mathrm{d}u \tag{A3}$$

The mission function is specified in this case as

$$2M = a_1\left\{I_r + \int_{l_0}^{l}\rho\left[v^2 - \frac{l^2-u^2}{2}\left(\frac{\partial v}{\partial u}\right)^2\right]\mathrm{d}u\right\}\left(\frac{\partial\theta}{\partial t}\right)^2 + a_2(\theta-\theta_m)^2 + (a_1+b)\left[\int_{l_0}^{l}\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial\theta}{\partial t}\right)^2\mathrm{d}u + \int_{l_0}^{l}EI\left(\frac{\partial^2 v}{\partial u^2}\right)^2\mathrm{d}u\right] \tag{A4}$$

Assuming that the bending rigidity EI and the damping coefficients c_1 and c_4 are constant throughout the beam, one obtains the derivative of the mission function with respect to time in the same form as Eq. (8) with the expression of the torque, however, in addition to a small term as follows:

$$T_r = (1/a_1)\Big\{\tilde{T} - a_2(\theta - \theta_m) + b(M_0 - l_0 S_0) - c_1[b^2/4(a_1+b)][(l^3 - l_0^3)/3](\partial\theta/\partial t) + c_4(b/EI)(\partial M_0/\partial t - l_0 \partial S_0/\partial t) - \frac{\partial\theta}{\partial t}\int_{l_0}^{l} \rho\left(\frac{\partial v}{\partial t} + 2u\frac{\partial\theta}{\partial t}\right) v \,\mathrm{d}u\Big\} \tag{A5}$$

Selection of the value of $\tilde{T}$ as in Eq. (11) gives the time derivative of the mission function in the same expression as Eq. (12). Thus, it is evident, even in this case with the inclusion of the higher-order terms into the equations of motion, that the mission state is asymptotically stable, and the control torque T_r will accomplish the mission if implemented through Eqs. (A5) and (11) as follows:

$$T_r = (1/a_1)\Big\{a_2(\theta_m - \theta) - k(\partial\theta/\partial t) + b(M_0 - l_0 S_0) - c_1[b^2/4(a_1+b)][(l^3 - l_0^3)/3](\partial\theta/\partial t) + c_4(b/EI)(\partial M_0/\partial t - l_0 \partial S_0/\partial t) - \frac{\partial\theta}{\partial t}\int_{l_0}^{l} \rho\left(\frac{\partial v}{\partial t} + 2u\frac{\partial\theta}{\partial t}\right) v \,\mathrm{d}u\Big\} \tag{A6}$$

It may be noted that the expression of T_r in Eq. (A6) includes information of the distributed data as $(\partial\theta/\partial t)\int_{l_0}^{l} \rho(\partial v/\partial t) + 2u(\partial\theta/\partial t)v\,\mathrm{d}u$ which is hard to implement. The additional term, however, is a high-order term, very small in its value, and negligible when the value of the attitude velocity $\partial\theta/\partial t$ is small enough.

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